Marathwada Institute of Technology, Aurangabad.
Department of Computer Science & Engineering
Year 2014-2015

Class: SE (CSE)
Subject: Discrete Mathematics
Subject In charge: Prof. L. B. Randive

Unit -1(SET THEORY)

2 Marks Questions

1. List the members of these sets
a. {x | x is number such that x2 =1}
b. {x | x is positive integer less than 12}
c. {x | x is the square of an integer and x < 100}
d. {x | x is an integer such that x2 = 2}

2. Use set builder notation to give description of each these sets.
a. {0, 3, 6, 9, 12}
b. {-3, -2, -1, 0, 1, 2, 3}
c. {m, n, o, p}
d. {2, 4, 6, 8, 10, 12, 14}

3. Determine whether each of these pairs of sets are equal.
a. {1, 3, 3, 3, 5, 5, 5, 5, 5}, {5, 3, 1}
b. {{1}},{1,{1}}
c. Φ, {Φ}

4. Find the power set of each of these sets
a. {a}
b. {a, b}
c. { Φ ,{ Φ }}
d. {a, b, c}
e. {a,b,c, Φ}

5. Let A= {a, b, c, d} and B={y, z}. Find a. A x B b. B x A

6. Determine whether each of these statements is true or false
a. $0 \in \Phi$
b. $\Phi \in \{0\}$
c. $\{0\} \subset \Phi$
d. $\Phi \subset \{0\}$
e. $\{\Phi\} \subset \{\Phi, \{\Phi\}\}$
f. $\{\Phi\} \in \{\{\Phi\}\}$

7. Let A={1, 2, 3, 4, 5} & B={0, 3, 6} find
a. $A \cup B$
b. $A \cap B$
c. A – B
d. B – A

8. Define with example:
i. Set
ii. Subset

iii. Power set
iv. Universal set
v. Conditional probability with example
vi. Symbols used for Venn Diagrams
vii. Cartesian product of two set
viii. Proper Subset

9. Explain in all operations on set with example.
10. State & prove DeMorgan's low of set theory.
11. Enlist the laws of set theory.
12. State and prove principle of Inclusion & Exclusion.

13. Write the following set in tabular form
    a. A= {x:x2=9}
    b. B={ x: x is multiple of 3 and 0<x<20}

14. Let A={ Φ , a} construct the following sets
a. { Φ}-A
b. $A \cap P(A)$

15. If A={n: n∈N,4 <n<12}
    B={n:n∈N,8<n<15} enlist elements of set A and B find $A \cup B$.

16. Define 1. Sample space 2. Mutually exclusive event.

17. Draw a Venn diagram of the set A, B, C for a given combination
    i. $A \cap (B \cap C)$
    ii. $A \cap (B \cup C)$
    iii. A-B
    iv. B-A
    v. $A \Delta B$
    vi. $A \Delta (B \Delta C)$

18. If we toss a fair coin, what is the probability that we will get a head?

19. If Satish rolls a fair die, what is the probability he gets
    i. a 5 or a 6
    ii. an even number
20. Use set builder notation to give description of each of these sets:
    1. {-3, -2,-1,0,1,2,3}
    2. {2,4,6,8,10,12,14}

7-8 Marks Questions

1. Let A, B, C be sets. Show that,
    a. $A \cup (B \cup C) = (A \cup B) \cup C$
    b. $A \cap (B \cap C) = (A \cap B) \cap C$
    c. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

2. Let A, B, C be sets. Show that,
    $(A - B) - C = (A - C) - (B - C)$

3. Show that if A & B are sets then, $(A \oplus B) \oplus B = A$

4. Show that,

a. $A \oplus B = (A \cup B) - (A \cap B)$

b. $A \oplus B = (A - B) \cup (B - A)$

5. Draw a Venn diagram for each of these combinations of the sets A, B & C

a. $A \cap (B \cup C)$

b. $(A - B) \cup (A - C) \cup (B - C)$

c. $A \cap B \cap C$

6. State & Prove

a. Associative Law.

b. Distributive Law.

c. De-Morgan‟s Law.

7. A person draw one card from a standard deck. If A, B, C denotes the events,

A : The card is spade

B : The card is red

C: The card is picture card. Find $Pr(A \cup B \cup C)$

8. For experiment of rolling a die, if the probability of occurrence of each sample is equal, then find the probability of getting an odd number.

9. The survey was conducted among 1000 people. Of course 595 are graduates, 595 wear glasses and 550 like ice-cream, 395 of them are graduates who wear glasses, 350 of them are graduates who like ice- cream and 400 of them wear glasses who like ice-cream. How many of them who are not graduates do not wear glasses and do not like ice-cream? How many of them are graduates do not wear glasses and do not like ice-cream?

10. In the survey of 60 people, it was found that 25 read Newsweek magazine, 26 read time, 26 read fortune. Also 9 read both Newsweek and fortune, 11 read both Newsweek and Time, 8 read both Time and fortune and 8 read no magazine at all.

i. Find out the no of people who read all the three Magazines.

ii. Fill in the correct numbers in all the regions of the Venn diagram.

iii. Determine number of people who reads exactly one Magazine.

12. For μ={1,2,3,…..,9,10} let A={1,2,3,4,5}, B={1,2,4,8} C={1,2,3,5,7}, and D={2,4,6,8}.
Determine each of following:

a. $(AUB) \cap C$

b. $A \cup (B \cap C)$

c. $C \cup D$

d. $C \cap D$

e. $(A \cup B) - C$

f. $(B - C) - D$

g. $A \cup (B - C)$

h. $(A \cup B) - (C \cap D)$

i. $B - (C - D)$

j. $A \Delta B$

13. A coin is loaded so that Pr(H) =2/3 and Pr (T) = 1/3.Todd tosses this coin twice. Let A, B be the events

A: the first toss is a tail.   B: Both tosses are the same. Are A, B independent?

14. At the high school science fair, 34 student received award for scientific projects. Fortune awards were given for projects in biology. 13 in chemistry, and 21 in physics each of the three student received awards in all three subject area, how many received awards for exactly

(a) one subject area? (b) Two subject area

15. Simplify the expression using laws of set theory.

$$\overline{\overline{(A\cup B)\cap C}\cap\overline{B}}$$

16. Prove the following using Venn diagram A Δ (B Δ C) = (A Δ B) Δ C

17. It was found that in first year of computer science of 80 students. 50 know COBOL, 55 know C, and 46 know Pascal. It was also known that 37 know C and COBOL, 28 know C and Pascal, and 25 know Pascal and COBOL. 7 students however know none of language. Find

I. How many know all the three languages?
II. How many know exactly two languages ?
III. How many know exactly one language?

18. Two dice are rolled together. Event A denotes that the sum of the number on the top faces is even and event B denotes that there is a 4 on at least one of the top faces. Find P(A∪B) and P(A∩B)

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Unit -2 Fundamentals of Logic

2 Marks Questions

1. Which of these sentences are propositions? What are the truth values of those that are propositions?
   a. Boston is the capital of Massachusetts
   b. Delhi is capital of India.
   c. 2 + 3 = 5
   d. 5 + 7 = 10
   e. X + 2 = 11
   f. Answer this question
   g. X + Y = Y + X for every pair of real numbers
   h. Do not pass go
   i. What is it
   j. 4 + X = 5

2. What is negation of each of these proposition
   a. Today is Thursday
   b. There is no pollution in New Jersey
   c. 2 + 1 = 3

3. Let p & q be the propositions
   p : I bought a lottery ticket this week
   q : I won the million dollar jackpot on Friday

Express each of these propositions as an English sentence
   a. $\neg p$
   b. $p \vee q$
   c. $p \rightarrow q$
   d. $p \wedge q$
   e. $p \leftrightarrow q$
   f. $\neg p \rightarrow \neg q$
   g. $\neg p \wedge \neg q$
   h. $\neg p \vee (p \wedge q)$
   4. Let p & q be the propositions
   p : It is below freezing q : It is snowing

4. Write these propositions using p & q and logical connectives.
   a. It is below freezing and snowing
   b. It is below freezing but not snowing
   c. It is not below freezing but not snowing d. If it is below freezing, it is also snowing.

5. State the converse, contra positive and inverse of each of these implications
   a. If it snows today, I will ski tomorrow
   b. I come to class whenever there is going be a quiz

c. A positive integer is a prime only if it has no divisors other than 1 and itself.

6. Find Dual of $(p \rightarrow q)$.

7. Define:
   1. Primitive statements.
   2. Truth table.
   3. Logical Equivalence.
   4. Principle of Duality.
   5. Tautology Contradiction.
   6. Logical Implication.

8. State rule of Modus Ponens with example.

7-8 Marks Questions

8. Constructor a truth table for each of these compound propositions
   a. $(p \land q) \rightarrow (p \lor q)$
   b. $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
   c. $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
   d. $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
   e. $(p \oplus q) \rightarrow (p \land q)$
   f. $(p \lor q) \rightarrow (p \oplus q)$

9. Let p, q, r denotes primitive statements
   a. Use truth table to verify the following logical equivalences
      1. $p \rightarrow (q \land r) \Longleftrightarrow (p \rightarrow q) \land (p \rightarrow r)$
      2. $[(p \lor q) \rightarrow r] \Longleftrightarrow [(p \rightarrow r) \land (q \rightarrow r)]$
      3. $[p \rightarrow (q \lor r)] \Longleftrightarrow [\neg r \rightarrow (p \rightarrow q)]$
   b. Use the substitution rule to show that
      $[p \rightarrow (q \lor r)] \Longleftrightarrow [(p \land \neg q) \rightarrow r]$

10. Verify the first Absorption law by means of truth table.

11. Verify the second Distributive law by means of truth table.

12. Determine whether $(\neg p \land (p \rightarrow q)) \rightarrow \neg q$ is a tautology

13. Show that each of these implications is a tautology by using truth table
    a. $[\neg p \land (p \lor q)] \rightarrow q$
    b. $[(p \rightarrow q) \land (q \rightarrow r)] \rightarrow (p \rightarrow r)$
    c. $[p \land (p \rightarrow q)] \rightarrow q$
    d. $[(p \lor q) \land (p \rightarrow r) \land (q \rightarrow r)] \rightarrow r$
14. Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not equivalent.

15. Consider the following argument.
    1) Rita is baking a Cake
    2) If Rita is baking a cake, then she is not practicing her flute.
    3) If Rita is not practicing her flute, then her father will not buy her a car.
    4) Therefore Rita‟s father will not buy her a car.
    Prove the validity of the argument using rules of inference.

16. Problems to simplify switching network.

17. What do you mean by logical connectives? Enlist all the logical connectives with example.

18. Enlist the lows of the logic.

19. Sate and explain the rules of inference with example

20. Let „p‟ be the proposition “high speed driving is dangerous” and „q‟ be the proposition “Rajesh was
a wise Man”. Write down meaning of the following proposition:

i. $P \wedge q$
ii. $\sim (p \wedge q)$
iii. $\sim p \wedge q$
iv. $(p \wedge q) v (\sim p \wedge \sim q$
v. $(pvq) \wedge \sim (p \wedge q)$

21. Construct truth tables to determine whether each of the following is a tautology , a contingency or a contradiction.

i. $P \rightarrow (q \rightarrow p)$

ii. $(p \wedge q) \wedge \sim (pvq)$

iii. $(p \wedge q) \rightarrow p$

iv. $(p \rightarrow q) \leftrightarrow (q \text{ v } \sim p)$

v. $(p \wedge (\sim pvq)) \wedge \sim q$

21. Determine whether the following is a valid argument:

i) If Geeta goes to class, she is on time but Geeta is late. She will therefore miss class.

ii). I am happy if my program runs. A necessary condition for the program to run is it should be error free. I am not happy. Therefore a program is not error free

22. Ramesh is studying ORACLE or he is not study JAVA. If Ramesh is studying JAVA, then he is not studying ORACLE. Therefore he is studying ORACLE. Write above statement in a symbolic form and test the validity of argument using laws of logic.

23. Test the validity of following statement:

If there is strike by student, then exam will be postponed.
Exam was not postponed.
Therefore there were no strikes by students

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Unit -3 Fundamentals of Logic Cont.

2 Marks Questions

1. Define rule of universal specification and generalization.

2. Define the sequence S(1)=1 and S(n)=2S(⌊n/2⌋) for n>=2. Find S(73) using Recursive Definition.

3. Define
   a. Open statement
   b. Existential Quantifier
   c. Universal Quantifier
   d. Logical implication.
   e. Contra positive, inverse & converse of open statement.
   f. Rule of universal Specification

4. Lets p(x) be the open statement "x2 = 2x" where the universe comprise all integers. Determine whether each of the following statement is true or false
   i. For some x p(x) ii. For all x p(x)
5. Let K(x): x is man L(x): x is mortal Mx): x is an integer N(x): Either positive or negative express the following using quantifier
   a. All men are mortal
   b. Any integer is either positive or negative
6. Negate the following
   $\exists x[r(x) \cap s(x)]$ where r(x) and s(x) are open statements.

7-8 Marks Questions

1. Translate each of these statements into logical expressions using predicates, quantifiers and logical connectives
   a. No one is perfect
   b. Not everyone is perfect
   c. All your friends are perfect
   d. One of your friends is perfect
   e. Everyone is your friend and is perfect
   f. Not everybody is your friend or someone is not perfect

2. If n is a positive integer, prove that 1.2 + 2.3 + 3.4 + …. + n(n+1)= n(n+1)(n+2)/3

3. Using mathematical induction

$$\sum_{i=1}^{n} i = 1+2+3+\ldots+n = n(n+1)/2$$

4. Let p(x), q(x), and r(x) be the following open statement.

p(x): x2 -7x + 10 = 0

q(x): x2 -2x - 3= 0

r(x): $x<0$

∃

5. Let p(x), q(x) and r(x) denote the following open statement.

p(x): x2- 8x +15=0 ∃

q(x): x is odd

r(x): $x>0$

For Q. no 4 and 5: For the universe of all integers, determine the truth or falsity of each of the following statements. If a statement is false, gives a counterexample.

i. $\forall x[p(x)\rightarrow q(x)]$

ii. $\forall x[q(x)\rightarrow p(x)]$

iii. $x[p(x)\rightarrow q(x)]$

iv. $x[q(x)\rightarrow p(x)]$ ∃

v. $x[r(x)\rightarrow p(x)]\exists$

∃

vi. $x[p(x)\rightarrow (q(x)\wedge r(x))]$

vii. $\forall x[(p(x) \vee q(x))\rightarrow r(x)]$

6 .Lets p(x) be the open statement “x2 = 2x” where the universe comprise all integers. Determine whether each of the following statement is true or false

i p(0) ii p(1) iii p(2) iv. p(-2)

7. Explain Quantifier. For the universe of all integers following are the open statements

p(x): $x>0$

q(x):x is even

r(x) : x is a perfect square

s(x): x is divisible by 4

t(x): x is divisible by 5

Write the following statement in symbolic form and determine whether each of the following statement is true or false. For each false statement provide a counter example

i) At least one integer is even
ii) There exist a positive integer that is even
iii) If x is even, then x is divisible by 5
iv) If x is even and x is perfect square then x divisible by 4.

8. Negate and simplify the following compound statement: $(p\vee q)\rightarrow r$

9. Prove by mathematical induction n^3+2n is divisible by 3 for all n>=3

10. Prove by using mathematical induction:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

11. Rewrite the following arguments using quantifiers, variables and predicate symbols
    1. All birds can fly
    2. Some men are genius.
    3. Not all birds can fly.
    4. There is a student who likes mathematics but not geography.

12. Define Well –ordering principle and mathematical induction theorem.

13. Define recursive definition. Prove that

$$\sum_{i=1}^{n} F_i^2 = F_n * F_{n+1}$$

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Unit -4 Relation and Function

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2 Marks Questions

1. If A = {1, 2, 3, 4}, B = {2, 5}, C = {3, 4, 7} determine
    a. A x B
    b. B x A
    c. A ∪ (B x C)
    d. (A ∪ B) x C
    e. (A x C) ∪ (B x C)

2. Define binary relation and explain its properties.

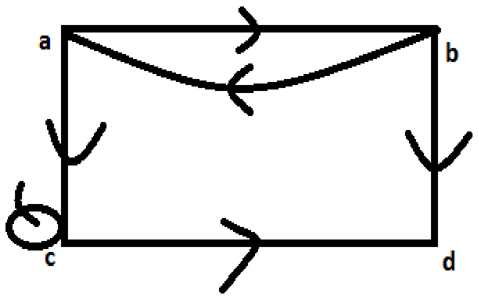
3. Define
    1. Partial order Relation
    2. Equivalence relation
    3. Composite function
    4. Bijective function

4. What is difference between onto and one-to-one function?

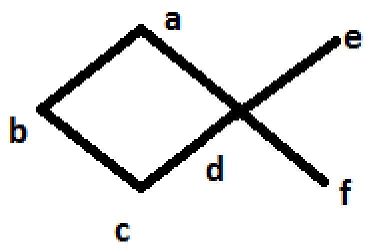
5. Let A={4,5,6,7} , relation R_1 is { (4,4),(5,4),(7,6),(6,7)}. Determine whether relation is reflexive or symmetric.

6. If A={2,3,4} and B={5,6}. Determine all function from A to B. Let R be a relation on the set A={1,2,3,4,5,6,7} defined by R={(a,b)$\in AXA: 4\ divides\ a-b$}
    a. list the elements of R
    b. find the range of R

7. State Pigeonhole principle and give example.
8. For the given digraph write the relation and zero-one matrix.



9. Consider the POSET whose Hasse diagram is given below.



Find maximal elements and given example. Find glb (a,c), lub(e,d).

10. If A={2,3,4} B={5,6} determine all function from A to B.

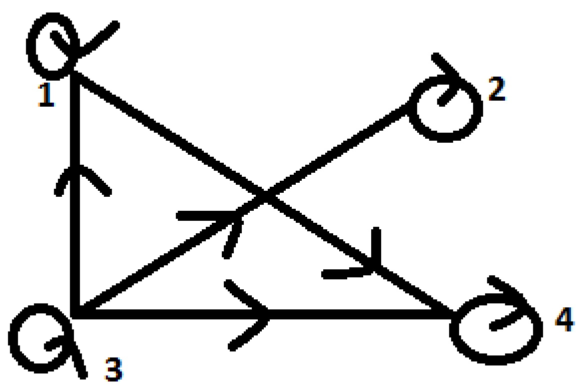
11. let R be an relation on set A={1,2,3,4,5,6,7} defined by R={(a,b)<-AXA:4 Divides(a-b)}
    a) List element of R
    b) Find the range of R

10) let A={7,8,9}. Determine all the partitions of the set.

7 - 8 Marks Questions

1) Let a={1,2,3,4,5,6,7}and R be the relation on set A R={(x,y)/(x-y)is divisible by3}
    a)show that R is an equivalence relation
    b) find R equivalence classes generated by elements of A

2) The directed graph G for a relation R on set A={1,2,3,4} is shown in fig.



    a) verify that (A,B) is POSET and find its Hasse diagram.
    b)Topologically sort (A,R)

3) define composite function. Function f,g are defined on a set x={1,2,3} as f={(1,2),(2,3),(3,1)} g={(1,2),(2,1),(3,3)} find fog , gof are they equal ?

4) Determine whether following relation is a function if YES find its range.
    a){(x,y)/x,yϵZ,y=x^2 } are in from Z to Z
    b){(x,y)/x,yϵR,y^2=x} a relation from R to R where R is set of real no.

5) Explain Pigeonhole principle with example

6) if A={1,2,3,4,5,6} and relation R={(1,1),(1,5),(2,2),(2,3),(2,6)

(3,2),(1,5),(3,6,(4,4),(5,1),(5,5),(6,2),(6,3),(6,6)}.
Show whether R is equivalence & relation and find equivalence classes of R.

7) What is chain and ant chain? Explain with example.

8) let A=B={1,2,3,4,5,12,15,25} and R be binary relation on set such that R={(a,b)/a divides b, ∀ a,bϵA}.show that R is partial order relation and draw Hasse diagram.

9)Let A=B{1,2,3,4}define f:A→B such that

1)f is one to one an onto
2) f is neither one to one nor onto
3) f is onto but not one to one.
4)f is one to one but not onto.

10) State pigeon hole principle show that if seven numbers from 1 to 12 are chosen then two of them will add up to 13.

11) Let f,g,h be function from Z to Z defined by f(x)=x-1 g(x)=3x

$$h(x)=\begin{cases} 0 \text{ if x is even} \\ 1 \text{ if x is odd} \end{cases}$$

Show that(fog)oh=fo(goh).

12). let {a}={1,2,3,4,6,8,12} and R be the partial order on A defined by aRb if a divided b

1.draw hasse diagram of poset (A,R)
2.determine the relational matrix for R
3.construct directed graph G on A
4.topologically sort the poset (A,R)

13. Explain converse of functional and invertible function.

14. Explain pigeon hole principle if five colors are used to paint 26 doors show that at least Six doors will have the same door

15. let f(x) =X+2, g(X)=x-2, h(x)=3x for bϵR where R is set of real nos
Find gof, foh, fohog, hoq.